Problem Set 4 – due Oct. 9

1. Use importance sampling and Monte Carlo integration to compute

$$\int_0^1 dx \, \frac{e^{-10x}}{1+x}$$

2. Using either your computer's random number generator or one of the random number subroutines on the web site, test the quality of the output. Specifically, check whether the output numbers fill the unit interval uniformly and whether they fill the plane uniformly when grouped in pairs.

3. (1-d Ising model) Consider a one-dimensional periodic lattice with N sites, each of which has a "spin" which can be up or down, $s_i = \pm 1$. Each spin interacts with its neighbors to each side with an energy which favors alignment of the spins – negative when both are up or both are down, and positive otherwise. The explicit form of the total energy is

$$E = -\epsilon \sum_{i=1}^{N} s_i s_{i+1}$$

and the periodicity is taken into account by identifying $s_{N+1} = s_1$.

Compute the average energy and the average spin of the lattice as a function of temperature using the Metropolis algoritm. Do this for $N = 100$ at temperatures $k_B T = \epsilon/2$ and 2ϵ. In each case, run the simulation until the average energy stabilizes.