Problem Set 2 – due Sept. 18

1. A zipper has N links, and each has a "closed" state with energy 0 and an "open" state with energy ϵ. Suppose the zipper can only open from the left end, and that link number s can be open only of all links to its left (1, 2, ..., s-1) are already open. Suppose the zipper floats in a liquid at temperature T, which acts as a heat bath.

   (a) Show that the partition function is

   $$Z = \frac{1 - \exp\left[-(N+1)\epsilon/kT\right]}{1 - \exp\left[-\epsilon/kT\right]}$$

   (b) Find the average number of open links in the two limits $\epsilon \gg kT$ and $\epsilon \ll kT$, and interpret the result.

2. Show that in the canonical ensemble the entropy is given by

   $$S = -k_B \sum_s p_s \log p_s$$

   where p_s is the probability of being in state s.

3. For some material, the entropy is related to the number of particles N, the energy E and the volume V by

   $$S = \lambda V^{1/2} (NE)^{1/4}$$

   where λ is a constant.

   (a) Find the temperature and pressure.

   (b) The material is placed in a box divided in half by a partition, with 1/3 of the particles and 1/3 of the energy in the left half and the remainder on the right. If the partition is fixed in place but conducts heat, what is the final distribution of energy between the two halves of the box after equilibrium is reached?

   (c) If the partition is allowed to slide back and forth (without friction) as well as conduct heat, what is the final distribution of energy *and volume*?