Physics V1100 **Analytical Dynamics** **Fall 2014**

Problem Set 7 – due November 5

1. In an inertial reference frame S', which moves relative to the lab frame S with velocity $\mathbf{u}$, a particle is observed to have velocity $\mathbf{v}'$ and acceleration $\mathbf{a}'$. Show that in S, the components of the particle's acceleration parallel and perpendicular to $\mathbf{u}$ are

$$\mathbf{a}_\parallel = \frac{\gamma^{-3}}{(1+\mathbf{u}\cdot\mathbf{v}')^3}\,\mathbf{a}_\parallel' \qquad \mathbf{a}_\perp = \frac{\gamma^{-2}}{(1+\mathbf{u}\cdot\mathbf{v}')^3}\,[\mathbf{a}_\perp' + \mathbf{u}\times(\mathbf{a}'\times\mathbf{v}')]$$

   (10 points)

2. Show that a finite boost matrix, $\mathbf{B}(\mathbf{u}) = \exp(-i\mathbf{u}\cdot\mathbf{K})$, can be expanded as

$$\mathbf{B}(\mathbf{u}) = \mathbf{1} - i(\mathbf{u}\cdot\mathbf{K})\sinh|\mathbf{u}| + (\mathbf{u}\cdot\mathbf{K})^2(1-\cosh|\mathbf{u}|)$$

   where $(\mathbf{K}_j)_{\mu\nu} = i(\delta_{j\mu}\delta_{0\nu} + \delta_{0\mu}\delta_{j\nu})$ is the boost generator. (10 points)

3. It was shown in class that any proper Lorentz transformation can be decomposed into a rotation followed by a boost, $\mathbf{\Lambda} = \mathbf{B}\cdot\mathbf{R}$. Show that $\mathbf{B}$ and $\mathbf{R}$ are unique. (5 points)

4. In the lab frame, a meter stick approaches the origin with velocity u close to c along the negative x-axis. A very thin plate parallel to the x-y plane with a circular hole 0.5 meters in diameter approaches the origin along the negative z-axis with velocity v, also close to c. Both arrive at the origin simultaneously (see figure). A lab observer might say that the meter stick is Lorentz contracted and will easily pass through the hole in the plate. An observer moving with the stick might instead say the the plate and hole are Lorentz contracted, while the stick is not, and the stick can't possibly pass through. Who is correct and why? (15 points)