Problem Set 6 – due October 29

1. An anisotropic harmonic oscillator with

$$V(\mathbf{r}) = \frac{1}{2}m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

has charge q and is placed in a uniform magnetic field $\mathbf{B}$ parallel to the z-axis. Find the normal modes of small oscillations about $\mathbf{r} = 0$. Discuss the result for the case $qB/mc \ll |\omega_i|$. Hint - use the Lagrangian for a particle in an electromagnetic field discussed in Section 1.5. (10 points)

2. Goldstein, Chapter 6, Exercise 10, page 273. (15 points)

3. Find the normal modes of vibration for the double pendulum shown in the figure below, with equal lengths but unequal masses. Show that if $m_1 \ll m_2$ and the upper mass is slightly displaced from the vertical and released, the two masses oscillate 90° out of phase. In the other limit, $m_2 \ll m_1$, and the same initial condition, show that at regular intervals one pendulum is at rest while the other has its maximum amplitude. (15 points)