Physics 71100 **Analytical Dynamics** **Fall 2014**

Problem Set 5 – due October 20

1. Goldstein, Chapter 5, Exercise 15, page 235 (10 points).
2. Goldstein, Chapter 5, Exercise 18, page 235 (10 points).
3. A rigid lamina (*i.e.*, a two-dimensional object) lies in the x-y plane.

   (a) Show that $\hat{e}_3 \equiv \hat{z}$ is a eigenvector of the inertia tensor and that its eigenvalue $I_3 \equiv I_{zz}$ is equal to the sum of the other two principal moments of inertia.

   (b) Suppose, in particular, that the principal moments of inertia about the center of mass are given by

   $$I_1 = (\mu^2 - 1), \quad I_2 = (\mu^2 + 1), \quad I_3 = 2\mu^2.$$

   Write down Euler's equations for the lamina moving freely in space, and show that the component of the angular velocity in the plane of the lamina, $(\omega_1^2 + \omega_2^2)^{1/2}$, is constant in time.

   (c) Choose the initial angular velocity to be $\boldsymbol{\omega} = N\mu\hat{e}_1 + N\hat{e}_3$ and define $\tan\alpha = \omega_2/\omega_1$, which is the angle the component of $\boldsymbol{\omega}$ in the plane of the lamina makes with $\hat{e}_1$. Show that it satisfies

   $$\ddot{\alpha} + N^2 \sin\alpha\, \cos\alpha = 0.$$

   Solve this differential equation and show that at time t,

   $$\boldsymbol{\omega}(t) = (N\mu\, \mathrm{sech} Nt)\, \hat{e}_1 + (N\mu\, \tanh Nt)\, \hat{e}_2 + (N \mathrm{sech} Nt)\, \hat{e}_3.$$

   (20 points)