

Problem Set 3 – due September 29

1. (a) Suppose the action of a system of N particles is invariant under the “screw displacement”

$$r_i \rightarrow r_i \quad \theta_i \rightarrow \theta_i + \Delta \quad z_i \rightarrow z_i + \alpha\Delta \quad i = 1, \dots, N$$

in cylindrical coordinates, where α is the screw’s pitch and $|\Delta| \ll 1$. What quantity is conserved?

- (b) By examining the symmetries of the action, identify the constants of the motion for the three-dimensional harmonic oscillator, where

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 - \frac{1}{2}k\mathbf{r}^2$$

- (c) Show explicitly from the equations of motion for the 3-d harmonic oscillator that the quantities

$$C_{ij} = \frac{1}{2}(m\dot{r}_i\dot{r}_j + kr_i r_j)$$

are constants of the motion. How many constants of the motion can there be, and how are the C_{ij} related to those found in part (b)?

(15 points)

2. Goldstein *et al.*, Problem 13, page 128. (10 points)
3. Goldstein *et al.*, Problems 21 and 22, page 130. (15 points)