2 #. lab frame : $u_i = (c\gamma_i, \underline{u}\gamma_i)$, $\gamma_i = (1 - u_i^2/c^2)^{-\frac{1}{2}}$

In rest frame of #1 : $\begin{cases} u_1 = (c, \underline{0}) \\ u_2 = (c\gamma, \underline{V}\gamma), \quad \gamma = \left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}} \end{cases}$

so $c^2\gamma = u_1 \cdot u_2$

$= \gamma_1\gamma_2 (c^2 - \underline{u}_1 \cdot \underline{u}_2)$

or ~~for V~~ $1 - V^2/c^2 = \left(1 - \frac{u_1^2}{c^2}\right)\left(1 - \frac{u_2^2}{c^2}\right) \frac{1}{(1 - \underline{u}_1\cdot\underline{u}_2/c^2)^2}$

$$\underline{V}^2 = c^2 \left[1 - \frac{(1 - u_1^2/c^2)(1 - u_2^2/c^2)}{(1 - \underline{u}_1\cdot\underline{u}_2/c^2)^2} \right]$$

$$= \frac{c^2}{(\text{''})^2} \left[\left(\cancel{1} - 2\frac{\underline{u}_1\cdot\underline{u}_2}{c^2} + \frac{(\underline{u}_1\cdot\underline{u}_2)^2}{c^4} \right) - \left(\cancel{1} - \frac{u_1^2}{c^2} - \frac{u_2^2}{c^2} + \frac{u_1^2 u_2^2}{c^4} \right) \right]$$

$$= \frac{1}{(\text{''})^2} \left[\frac{(\underline{u}_1 - \underline{u}_2)^2}{c^2} - \underbrace{\frac{u_1^2 u_2^2 - (\underline{u}_1\cdot\underline{u}_2)^2}{c^2}}_{} \right]$$

$$= \frac{u_1^2 u_2^2}{c^2}(1 - \cos^2\theta)$$

$$= \frac{1}{c^2}(\underline{u}_1 \times \underline{u}_2)^2$$

3. (a) Without acceleration, motion of ball is

$$z(t) = h - \frac{1}{2} g t^2 \qquad -\sqrt{\frac{2h}{g}} \le t \le +\sqrt{\frac{2h}{g}}$$

so action variable is

$$J = \frac{1}{2\pi} \oint p\,dq = \frac{1}{2\pi} \cdot 2 \cdot \int_0^{\sqrt{\frac{2h}{g}}} dt \cdot m\dot{z}^2$$

$$= \frac{m}{\pi} \int_0^{\sqrt{\frac{2h}{g}}} dt\,(gt)^2 = \frac{m}{\pi} g^2 \frac{1}{3} \left(\frac{2h}{g}\right)^{3/2}$$

$$= \text{const.} \left(g^{\frac{1}{3}} h\right)^{3/2}$$

If the acceleration of the elevator is a, then $g \to g - a$, & if a varies slowly $J \simeq$ const.,

so $h \propto g^{-\frac{1}{3}}$ or $h(a) = h(0) \left(\frac{g-a}{g}\right)^{-\frac{1}{3}}$

(b) A constant velocity is completely irrelevant (by Galilean relativity).

4. $$L = e^{\lambda t/m}\left[\tfrac{1}{2} m \dot{q}^2 - \tfrac{1}{2} m \omega^2 q^2\right]$$

$$p = \frac{\partial L}{\partial \dot{q}} = m \dot{q}\, e^{\lambda t/m}$$

$$H = p\dot{q} - L = \left(\tfrac{1}{2} m \dot{q}^2 + \tfrac{1}{2} m \omega^2 q^2\right) e^{\lambda t/m}$$

$$= \frac{p^2}{2m} e^{-\lambda t/m} + \tfrac{1}{2} m \omega^2 q^2 e^{+\lambda t/m}$$

if $F_2 = q P e^{\lambda t/2m}$

then $$Q = \frac{\partial F_2}{\partial P} = q e^{\lambda t/2m}, \quad p = \frac{\partial F_2}{\partial q} = P e^{\lambda t/2m}$$

$$K = H(Q, P) + \frac{\partial F_2}{\partial t}$$

$$= P^2/2m + \tfrac{1}{2} m \omega^2 Q^2 + \frac{\lambda}{2m} Q P$$

$$\dot{P} = -\frac{\partial K}{\partial Q} = -m \omega^2 Q - \lambda P/2m = [P, K]$$

$$\dot{Q} = \frac{\partial K}{\partial P} = P/m + \lambda Q/2m = [Q, K]$$

$$\Rightarrow \ddot{Q} = \dot{P}/m + \lambda \dot{Q}/2m$$

$$= -\omega^2 Q - \lambda P/2m^2 + \frac{\lambda}{2m}\left(P/m + \lambda Q/2m\right)$$

$$= -\left(\omega^2 - \lambda^2/4m^2\right) Q = -\tilde{\omega}^2 Q$$

$$Q(t) = A e^{i\tilde{\omega} t} + B e^{-i\tilde{\omega} t}, \quad q(t) = e^{-\lambda t/m} Q(t).$$

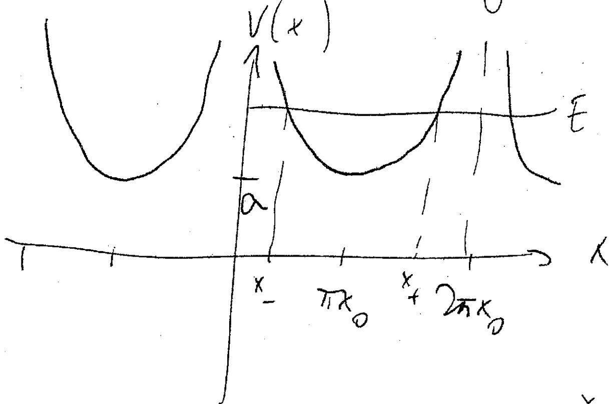
Goldstein 10.15

particle in 1-d with $V(x) = a \csc^2\left(\frac{x}{x_0}\right)$

HJ eqn is $\frac{1}{2m}\left(\frac{dW}{dx}\right)^2 + a\csc^2\frac{x}{x_0} = \alpha_1 \equiv E$

$\rightarrow \quad W = \int dx \sqrt{2m\left(E - a\csc^2\frac{x}{x_0}\right)}$

A-A variables good for periodic motion: true here for $E > a$



$x_-(E) < x < x_+(E)$

$$J = 2\int_{x_-(E)}^{x_+(E)} \sqrt{2m\left(E - a\csc^2\frac{x}{x_0}\right)}$$

$$\nu^{-1} = \frac{\partial J}{\partial E} = \sqrt{2m}\int_{x_-}^{x_+} dx \left(E - a\csc^2\frac{x}{x_0}\right)^{-\frac{1}{2}}$$

$$= \sqrt{2m}\int_{x_-}^{x_+} \frac{dx \sin x/x_0}{\sqrt{E\sin^2\frac{x}{x_0} - a}} = \pi x_0\sqrt{\frac{2m}{E}}$$

6. $\mathcal{L} = \frac{1}{2}\psi_x \psi_t + \frac{\alpha}{6}\psi_x^{3} - \frac{\nu}{2}\psi_{xx}^{2}$

$$\frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \psi_t} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial \psi_x} + \frac{\partial^2}{\partial x^2}\frac{\partial \mathcal{L}}{\partial \psi_{xx}} = 0$$

$$0 = 0 - \frac{\partial}{\partial t}\left(\frac{1}{2}\psi_x\right) - \frac{\partial}{\partial x}\left(\frac{1}{2}\psi_t + \frac{\alpha}{2}\psi_x^{2}\right) + \frac{\partial^2}{\partial x^2}\left(-\nu\psi_{xx}\right)$$

$$= -\psi_{xt} \mp \alpha\psi_x\psi_{xx} \mp \nu\psi_{xxxx}$$

let $\varphi = \psi_x$: $\quad 0 = \varphi_t + \alpha\varphi\varphi_x + \nu\varphi_{xxx}$.