

Second Examination – November 24 – 33 points per problem

1. In an inertial frame  $S$  a relativistic particle of mass  $m$  experiences a force  $F_x = -kx$ . If the particle starts at rest at  $x = a$ , find its velocity relative to  $S$  as a function of  $x$ .
2. A mass  $m$  is attached to a massless spring of stiffness  $k$  which hangs from the ceiling, and a second mass  $m$  is attached to the first by an identical spring. The entire system moves vertically. Find the normal modes and frequencies of small oscillations about equilibrium.
3. Lagrange's equations have been shown to be invariant under a "point transformation" of generalized coordinates,  $q_i \rightarrow Q_i(\mathbf{q})$ . What is the corresponding change in momenta  $p_i \rightarrow P_i(\mathbf{q}, \mathbf{p})$  required to make the transformation canonical?