Physics 71100 **Analytical Dynamics** **Fall 2014**

First Examination – October 8

Do any three of the four questions, 33 points per problem

1. Show that geodesics on the surface of a sphere – curves of minimum distance separating two points – are arcs of great circles.

2. A compound pendulum consists of a rod of length L and mass M pinned at one end, with the other end attached to the circumference of a uniform disk of radius a and mass m. The rod and disk move in a vertical plane, and both can swing freely about the points of attachment without friction. Find the equations of motion. Define precisely any moment of inertia you use, but you need not give its explicit value.

3. N particles of mass m move in 3d under the action of pairwise potentials,

$$V = \sum_{i<j} V_2(\mathbf{r}_i - \mathbf{r}_j)$$

subject to the constraint that their center of mass velocity is fixed,

$$\frac{1}{N}\sum_{i=1}^{N} \dot{\mathbf{r}}_i(t) = \mathbf{U}(t)$$

Using the constrained Lagrangian formalism, find the equations of motion in terms of the potential energy and $\mathbf{U}$, and from your result verify that the constraint is satisfied.

4. A particle of mass m moves in a central force field with potential $V(r)$. Find the conditions on V for which stable circular orbits about the force center are possible, and find the frequency of small oscillations about such an orbit.